Farey series

\[
\begin{align*}
t & \quad \text{IS} \quad \$255 \\
\text{LOC} & \quad \text{Data\_Segment} \\
_x & \quad \text{GREG} \quad @ \\
\text{LOC} & \quad \text{Data\_Segment+4*10000} \\
y & \quad \text{GREG} \quad @ \\
\text{LOC} & \quad \#100 \\
\end{align*}
\]

> Computation of Farey series

> Calling sequence: SET $1,n; \text{PUSHJ }$0,:Farey

> Entry conditions: \( n \) in $1$ is the order of the series, \( 1 < n < 182 \)

> \( _x \) and \( _y \) are arrays, each for at least 10000 tetras

> Exit conditions: $0$ is number of \( x/y \) pairs, i.e., entries generated in \( _x \) and \( _y \)

PREFIX :FAREY:

01 \( n \) IS $1$

Parameter: order of the Farey series

02 \( kk \) IS $2$

\( kk \leftarrow 4 \times k \)

03 \( yk \) IS $3$

\( yk \)

04 \( yk1 \) IS $4$

\( yk+1 \)

05 \( xk \) IS $5$

\( x_k \)

06 \( xk1 \) IS $6$

\( x_{k+1} \)

07 \( flr \) IS $7$

08 \( yk2 \) IS $8$

\( y_{k+2} \)

09 \( xk2 \) IS $9$

\( x_{k+2} \)

10 :Farey SET \( n,\$0 \)

1 Get the parameter.

11 SET \( xk,0 \)

1 Init for \( k = 0 \).

12 STTU \( xk,:_x,4\times0 \)

1

13 SET \( yk,1 \)

1

14 STTU \( yk,:_y,4\times0 \)

1

15 SET \( xk1,1 \)

1 Init for \( k = 1 \).

16 STTU \( xk1,:_x,4\times1 \)

1

17 SET \( yk1,n \)

1

18 STTU \( yk1,:_y,4\times1 \)

1

19 SET \( kk,4 \)

1 \( k \leftarrow 1 \).

20 nextval ADDU \( flr,yk,n \)

A Calculate the next values \( xk2 \) and \( yk2 \).

21 DIVU \( flr,flr,yk1 \)

A \( flr \leftarrow \left\lfloor \frac{(yk+n)}{yk1} \right\rfloor \).

22 MULU \( xk2,flr,xk1 \)

A

23 SUBU \( xk2,xk2,xk \)

A \( xk2 \leftarrow flr \times xk1 - xk \).

24 MULU \( yk2,flr,yk1 \)

A

25 SUBU \( yk2,yk2,yk \)

A \( yk2 \leftarrow flr \times yk1 - yk \).

26 INCL \( kk,4 \)

A \( k \leftarrow k + 1 \).

27 STTU \( xk2,:_x,kk \)

A

28 STTU \( yk2,:_y,kk \)

A

29 SET \( xk,xk1 \)

A Shuffle the registers.

30 SET \( xk1,xk2 \)

A

31 SET \( yk,yk1 \)

A

32 SET \( yk1,yk2 \)

A

33 CMPU \( flr,xk2,yk2 \)

A The computation stops when

34 PBNZ \( flr,nextval \)

A \( 1 = 1/1 \) is computed.

35 INCL \( kk,4 \)

1

36 SR \( kk,kk,2 \)

1 Remove factor for tetra.

37 SET \( \$0,kk \)

1 The number of elements

38 POP \( 1,0 \)

1 is returned.
Analysis

The subroutine :Farey costs \((2A + 4)\mu + (92A + 18)\nu\).

Let the length of a Farey series of order \(n\) be \(f_n\). Then the following relation holds: \(A = f_n - 2\). The value of \(f_1\) is 2 as there are just the two entries 0/1 and 1/1. The Farey series of order 2 has one more element as the only quotient to be added is 1/2. So \(f_2 = 3\). In general the step from \(f_{n-1}\) to \(f_n\) adds all quotients of the form \(x/n\) in which the \(x\) is relatively prim to \(n\). So \(f_n = f_{n-1} + \varphi(n)\). Therefore \(A = 2 + \varphi(2) + \varphi(3) + \cdots + \varphi(n) - 2\).

For test runs with \(n = 7, 13,\) and 39 the answers are \(f_7 = 19, f_{13} = 59,\) and \(f_{39} = 475\). So in the subroutine the value of \(A\) has to be \(17 + 57 + 473 = 547\).

The first call to Farey starts with 7 instructions, 1 mem, 11 oops; 0 good guesses, 0 bad and ends with 276 instructions, 39 mems, 1593 oops; 16 good guesses, 1 bad. Therefore the subroutine needs \(38\mu\) and \(1582\nu\). The second and third calls have \(118\mu + 5262\nu\) and \(950\mu + 43534\nu\). The measured data agree with the above stated cost function.